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SHOCK-WAVE STABILITY FOR ONE MODEL OF RADIATION HYDRODYNAMICS

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Various models of radiation hydrodynamics (including the models considered by Anile et al. in [1–4]) are widely used to describe real processes in some fields of physics such as astrophysics, cosmology, and plasma physics. A mathematical model of radiation hydrodynamics is usually a system of quasi-linear differential equations in the form of conservation laws (i.e., in divergent form) describing the interaction between a continuous medium and radiation [1].

In the case of a motionless continuum (precisely this case is considered herein), the equations of radiation hydrodynamics describing radiation propagation in the medium are derived, as is known, from the equations of transfer by the usual physical reasoning [2]. The resulting problem of closure is solved by introducing the Eddington factor (a thermodynamic approach to introducing this factor is described by Anile et al. in [2–4]).

In this paper, according to the approach of [5], we consider the stability of strong discontinuities (by analogy with gas dynamics, we call them shock waves) in a model of radiation hydrodynamics.

1. Radiation Hydrodynamics Equations and Their Symmetrization. Following [1, 2], we write the system of equations of radiation hydrodynamics in the form of laws of conservation of "mass" and "momentum" for the radiation energy:

$$\partial J/\partial t + \mathrm{div}\mathbf{H} = -\rho_0 æ(J - B); \tag{1.1}$$

$$\partial \mathbf{H}/\partial t + \mathrm{div}\mathcal{K} = -\rho_0 æ\mathbf{H}. \tag{1.2}$$

Here J is the radiation-energy density; $\mathbf{H} = (H^1, H^2, H^3)^* = J\Lambda = J(\Lambda^1, \Lambda^2, \Lambda^3)^*$ is the radiation-energy flux (the asterisk means transposition); $\mathcal{K}$ is the radiation stress tensor with the components

$$K^{ij} = J\left\{\delta^{ij}\varphi_1/3 + \varphi_2\Lambda^i\Lambda^j\right\}, \quad i,j = 1,\ 2,3; \tag{1.3}$$

$\varphi_1 = 1 - \varphi$; $\varphi_2 = \varphi/\lambda$; $\varphi = \varphi(\lambda) = (2 - \sqrt{4 - 3\lambda})$ is the modified Eddington factor [2]; $\lambda = |\Lambda|^2$; $|\Lambda|^2 = (\Lambda, \Lambda)$; $(\ ,\ )$ is the scalar vector product; ρ_0 is the density of the continuous medium through which radiation propagates; $æ$ is the absorption coefficient [1, 2] (below, without loss of generality, we assume that $\rho_0 æ \equiv 1$); $B = B(t, \mathbf{x})$ is a source function; $\mathbf{x} = (x^1, x^2, x^3)$; x^k ($k = 1, 2, 3$) are the Cartesian coordinates; the speed of light is considered equal to unity. By virtue of this, the inequalities [1, 2]

$$0 \leqslant \lambda < 1 \tag{1.4}$$

are valid for the parameter λ. Equations (1.1) and (1.2) can be regarded as a system for determining, for example, the components of the vector of the unknowns

$$\mathbf{U} = \begin{pmatrix} J \\ \mathbf{H} \end{pmatrix}.$$

Let us consider the question of symmetrization of Eqs. (1.1)–(1.2), i.e., the possibility of writing them as a symmetric t-hyperbolic (by Friedrichs) system (the symmetrization problem for the equations of continuum

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mechanics was considered in detail by Blokhin [5, 6]). As is known, when there is an additional conservation law for some system of conservation laws, the symmetrization problem is solved fairly simply (the symmetrization scheme for this case is described in detail [5, 6]). In our case, we shall follow this scheme [although an additional conservation law for system (1.1) and (1.2) is unknown]. We assume that the entropy law [1–4] is valid for Eqs. (1.1) and (1.2), i.e., that there are smooth functions $\Phi^{(\alpha)} = \Phi^{(\alpha)}(\mathbf{U})$, $r_\alpha = r_\alpha(\mathbf{U})$, $\alpha = \overline{0,3}$, and $g = g(\mathbf{U})$ such that the relation

$$r_0\left(\frac{\partial J}{\partial t} + \operatorname{div}\mathbf{H}\right) + \left(\mathbf{r}, \frac{\partial \mathbf{H}}{\partial t} + \operatorname{div}\mathcal{K}\right) = -r_0(J - B) - (\mathbf{r}, \mathbf{H}) = g = \frac{\partial \Phi^{(0)}}{\partial t} + \operatorname{div}\mathbf{\Phi} \tag{1.5}$$

holds for any smooth solution of system (1.1) and (1.2). Here $\mathbf{r} = (r_1, r_2, r_3)^*$; $\mathbf{\Phi} = (\Phi^{(1)}, \Phi^{(2)}, \Phi^{(3)})^*$; the functions $\Phi^{(\alpha)}$, r_α $(\alpha = \overline{0,3})$, and g depend in a smooth manner on the components of the vector $\mathbf{U}$ (the functions r_α $(\alpha = \overline{0,3})$ are also called canonical variables or Lagrangian multipliers [5, 6]).

We define the productive functions [5, 6] L and $M^{(k)}$ by the formulas

$$L = r_0 J + (\mathbf{r}, \mathbf{H}) - \Phi^{(0)}, \quad M^{(k)} = r_0 H^k + \sum_{i=1}^{3} r_i K^{ik} - \Phi^{(k)}, \quad k = 1,\ 2, 3. \tag{1.6}$$

According to the ideas given by Anile et al. in [1], we assume that

$$L = -\tilde{L}(G) r_0, \quad M^{(k)} = \tilde{L}(G) r_k, \quad k = 1,\ 2,\ 3. \tag{1.7}$$

Here $\tilde{L} = \tilde{L}(G)$ is a function to be defined, and $G = |\mathbf{r}|^2 - r_0^2$. It follows from (1.5) and (1.6) that

$$dL = J dr_0 + (\mathbf{H}, d\mathbf{r}), \quad dM^{(k)} = H^k dr_0 + \sum_{i=1}^{3} K^{ik} dr_i,$$

i.e., taking assumption (1.7) into account, we have

$$J = \partial L/\partial r_0 = 2r_0^2 \tilde{L}' - \tilde{L}, \quad \tilde{L}' = d\tilde{L}/dG; \tag{1.8}$$

$$H^k = \partial L/\partial r_k = \partial M^{(k)}/\partial r_0 = -2r_0 r_k \tilde{L}', \quad k = 1,\ 2, 3; \tag{1.9}$$

$$K^{ik} = \partial M^{(k)}/\partial r_i = 2r_i r_k \tilde{L}' + \tilde{L}\delta^{ik}, \quad i, k = 1,\ 2, 3. \tag{1.10}$$

Next, we assume that

$$r_0 = c_0/d, \quad \mathbf{r} = \mathbf{H}/d, \tag{1.11}$$

where the parameters c_0 and d satisfy the obvious relation

$$J^2\lambda = d^2 G + c_0^2 \quad (\lambda = |\Lambda|^2 = |\mathbf{H}|^2/J^2). \tag{1.12}$$

Comparing formulas (1.3) and (1.10), we obtain successively

$$2\tilde{L}' = \varphi_2/Jd^2; \tag{1.13}$$

$$\tilde{L} = \varphi_1 J/3. \tag{1.14}$$

From (1.9), (1.11), and (1.13) we find

$$c_0 = -J/\varphi_2. \tag{1.15}$$

With this choice of $\tilde{L}'$, $\tilde{L}$, and c_0, relation (1.8) holds identically. At the same time, taking into account (1.12), from (1.13)–(1.15) we have an ordinary differential equation for the unknown function $\tilde{L}$: $G\tilde{L}' = -2\tilde{L}$. We assume that one of its solutions is $\tilde{L} = 1/G^2$. Since

$$G = |\mathbf{r}|^2 - r_0^2 = -\frac{4}{3}\frac{J^2\varphi_1}{d^2\varphi_2} < 0,$$

the parameters G and d are given by the formulas

$$G = -\sqrt{\frac{3}{J\varphi_1}}, \qquad d = \sqrt{\frac{4J}{\varphi_2}\left(\frac{J\varphi_1}{3}\right)^{3/2}}. \tag{1.16}$$

With this choice of G, d, $\widetilde{L}$, and r_α $(\alpha = \overline{0,3})$, the functions $\Phi^{(\alpha)}$ $(\alpha = \overline{0,3})$ and g take the form

$$\Phi^{(0)} = \frac{4}{3} J r_0 \varphi_1, \quad \Phi^{(k)} = -\frac{2}{3} J r_k \varphi_1, \quad k = 1,2,3, \quad g = \frac{J}{d\varphi_2}(\varphi_1 J - B). \tag{1.17}$$

Formulas (1.16) and (1.17) complete the description of the additional conservation law.

Thus, the canonical variables r_α $(\alpha = \overline{0,3})$ and the productive functions L and $M^{(k)}$ $(k = 1, 2, 3)$ are defined. Then, following [5, 6], we write the initial system of equations of radiation hydrodynamics (1.1) and (1.2) as the following symmetric t-hyperbolic (by Friedrichs) system:

$$A^{(0)} \frac{\partial \mathbf{R}}{\partial t} + \sum_{k=1}^{3} A^{(k)} \frac{\partial \mathbf{R}}{\partial x^k} + D\mathbf{R} = \mathbf{F}. \tag{1.18}$$

Here $A^{(0)} = (L_{r_\alpha r_\beta}) = (4/G^4)\big(a^{(0)}_{\alpha\beta}\big)$; $A^{(k)} = \big(M^{(k)}_{r_\alpha r_\beta}\big) = (4/G^4)\big(a^{(k)}_{\alpha\beta}\big)$; $(\alpha, \beta = \overline{0,3};\ k = 1, 2, 3)$, $a^{(0)}_{00} = -3r_0(G + 2r_0^2)$; $a^{(0)}_{ii} = r_0(G - 6r_i^2)$; $a^{(0)}_{0i} = a^{(0)}_{i0} = r_i(G + 6r_0^2)$; $a^{(0)}_{ij} = a^{(0)}_{ji} = -6r_0 r_i r_j$; $a^{(k)}_{00} = r_k(G + 6r_0^2)$; $a^{(k)}_{0i} = a^{(k)}_{i0} = r_0(G\delta_{ik} - 6r_k r_i)$; $a^{(k)}_{ij} = a^{(k)}_{ji} = 6r_k r_i r_j - G(r_k\delta_{ij} + r_j\delta_{ik} + r_i\delta_{jk})$ $(i, j = \overline{1,3})$; $\mathbf{R} = (r_0, r_1, r_2, r_3)^*$; $\mathbf{F} = (B, 0, 0, 0)^*$; $D = d\,\mathrm{diag}\,(-\varphi_2, 1, 1, 1)$ is a diagonal matrix; $A^{(\alpha)}$ are symmetric matrices; $A^{(\alpha)} = (A^{(\alpha)})^*$ $(\alpha = \overline{0,3})$; and $A^{(0)} > 0$ if the natural condition $J > 0$ is satisfied.

Going back from the vector $\mathbf{R}$ to the vector $\mathbf{U}$, symmetric system (1.18) can also be rewritten. Actually, since $\mathbf{L_R} = \mathbf{U}$ and $d\mathbf{L_R} = A^{(0)}d\mathbf{R} = d\mathbf{U}$, i.e., $d\mathbf{R} = (A^{(0)})^{-1}d\mathbf{U}$, it follows from (1.18) that

$$B^{(0)} \frac{\partial \mathbf{U}}{\partial t} + \sum_{k=1}^{3} B^{(k)} \frac{\partial \mathbf{U}}{\partial x^k} + B^{(0)}\mathbf{U} = B^{(0)}\mathbf{F}, \tag{1.19}$$

where $B^{(0)} = B^{(0)}(\mathbf{U}) = m(A^{(0)})^{-1}$; $m = -4r_0 G_1/G^2$; $G_1 = 2r_0^2 - G$; $B^{(k)} = B^{(k)}(\mathbf{U}) = (1/m)\, B^{(0)} A^{(k)} B^{(0)}$ $(k = 1, 2, 3)$; and $B^{(\alpha)}$ $(\alpha = \overline{0,3})$ are symmetric matrices with $B^{(0)} > 0$.

2. Various Representations of System (1.1) and (1.2). Conditions on Strong Discontinuity. In system (1.1) and (1.2) we make the substitution

$$\mathbf{U} = \mathbf{U}' \mathrm{e}^{-t}. \tag{2.1}$$

Here

$$\mathbf{U}' = \begin{pmatrix} J' \\ \mathbf{H}' \end{pmatrix}$$

is the vector of the new dependent variables. Omitting the primes, for new variables we obtain

$$\partial J/\partial t + \mathrm{div}\, \mathbf{H} = \mathrm{e}^t B; \tag{1.1'}$$

$$\partial \mathbf{H}/\partial t + \mathrm{div}\, \mathcal{K} = 0. \tag{1.2'}$$

Let $B \equiv 0$. Then system (1.1′) and (1.2′) has the constant solution

$$\mathbf{U}(t, \mathbf{x}) = \hat{\mathbf{U}} = \begin{pmatrix} \hat{J} \\ \hat{\mathbf{H}} \end{pmatrix},$$

where $\hat{\mathbf{H}} = (\hat{H}^1, \hat{H}^2, \hat{H}^3)^*$; $\hat{J}$ and $\hat{H}^k$ $(k = 1, 2, 3)$ are constants. By virtue of (2.1), for $B \equiv 0$ initial system (1.1) and (1.2) has the solution

$$\mathbf{U} = \hat{\mathbf{U}} \mathrm{e}^{-t}. \tag{2.2}$$

In what follows, we need a nondivergent representation of the initial system of equations of radiation hydrodynamics. We introduce the "pressure"

$$p = (1/3) J \varphi_1. \tag{2.3}$$

Then, we have

$$dp = (1/3)\varphi_1 dJ - (J/\sigma)(\mathbf{\Lambda}, d\mathbf{\Lambda}) \qquad (\sigma = \sqrt{4 - 3\lambda}). \tag{2.4}$$

In view of (2.3) and (2.4), system (1.1′) and (1.2′) takes the form ($B \equiv 0$)

$$\frac{d''p}{dt} + p\,\mathrm{div}\boldsymbol{\Lambda} + \frac{1}{\sigma}p\,\mathrm{div}\,(\varphi\boldsymbol{\Lambda}) - \frac{1}{2\sigma}p\varphi_2(\boldsymbol{\Lambda},\nabla)\lambda = 0; \tag{2.5}$$

$$J\frac{d'\boldsymbol{\Lambda}}{dt} + \nabla p - \boldsymbol{\Lambda}\mathrm{div}\,(p\varphi_2\boldsymbol{\Lambda}) = 0. \tag{2.6}$$

Here $d'/dt = \partial/\partial t + \varphi_2(\boldsymbol{\Lambda},\nabla)$ and $d''/dt = \partial/\partial t + (1/\sigma)(\boldsymbol{\Lambda},\nabla)$.

For system (1.1′) and (1.2′) we write strong-discontinuity conditions. Let the surface of a strong discontinuity be given by the equation

$$\tilde{f}(t,\mathbf{x}) = f(t,\mathbf{x}') - x^1 = 0, \quad \mathbf{x}' = (x^2, x^3). \tag{2.7}$$

Then strong-discontinuity relations (2.7) take the form [7]

$$f_t[J] - [H^1] + \sum_{k=2}^{3} f_{x^k}[H^k] = 0; \tag{2.8}$$

$$f_t[H^1] - \left[p + \frac{\varphi_2}{J}(H^1)^2\right] + \sum_{k=2}^{3} f_{x^k}\left[\frac{\varphi_2}{J}H^1H^k\right] = 0; \tag{2.9}$$

$$f_t[H^i] - \left[\frac{\varphi_2}{J}H^1H^i\right] + f_{x^i}[p] + \sum_{k=2}^{3} f_{x^k}\left[\frac{\varphi_2}{J}H^iH^k\right] = 0, \quad i = 2,3, \tag{2.10}$$

where $[F]$ is the jump of F:

$$[F] = F^+ - F^-,$$

[F^+ and F^- are the values of F to the right ($-\tilde{f} \to +0$) and to the left ($-\tilde{f} \to -0$) of the surface of discontinuity (2.7)]. We shall use F and F_∞ in place of F^+ and F^-.

Considering (2.8)–(2.10), we can describe the following piecewise constant solution of system (1.1′) and (1.2′) for $B \equiv 0$ [piecewise smooth solution for system (1.1) and (1.2), see (2.1) and (2.2)]: for $x^1 < 0$

$$J = \hat{J}_\infty = \text{const} > 0, \quad H^1 = \hat{H}^1_\infty = \text{const}, \quad H^2 = H^3 = 0$$

(for definiteness, we assume that $\hat{H}^1_\infty > 0$); for $x^1 > 0$,

$$J = \hat{J} = \text{const} > 0, \quad H^1 = \hat{H}^1 = \text{const}, \quad H^2 = H^3 = 0,$$

and, for $x^1 = 0$, equalities (2.8)–(2.10) are satisfied (provided that the discontinuity front is motionless and is given by the equation $x^1 = 0$):

$$\hat{H}^1 = \hat{H}^1_\infty\,(= h) \neq 0; \tag{2.11}$$

$$[\hat{p}] + h^2[\hat{\varphi}_2/\hat{J}] = 0. \tag{2.12}$$

Here $[\hat{p}] = \hat{p} - \hat{p}_\infty$; $\hat{p} = (1/3)\hat{J}\hat{\varphi}_1$; $\hat{p}_\infty = (1/3)\hat{J}_\infty\hat{\varphi}_{1\infty}$; $\hat{\varphi}_1 = \hat{\sigma} - 1$; $\hat{\sigma} = \sqrt{4 - 3\hat{\lambda}}$; $\hat{\lambda} = (\hat{\Lambda}^1)^2 = h^2/\hat{J}^2$, etc.

By analogy with ordinary gas dynamics [5, 7], the stationary discontinuity described above is called a shock wave. In this case, relation (2.12) is an analog of the Hugoniot adiabat in ordinary gas dynamics. Rewriting relation (2.12) as

$$[\hat{J}(5 - 2\hat{\sigma})] = 0, \tag{2.12′}$$

we treat this equality as an equation for the parameter $k = \hat{J}_\infty/\hat{J}$. After simple calculations, we find roots $k_1 = 1$ and $k_2 = 9/(1 + 20\hat{\varphi}_\infty)$. Thus, for $1 > \hat{\varphi}_\infty > 2/5$ ($12/25 < \hat{\lambda}_\infty < 1$) the radiation-energy density increases in going through the shock-wave front (since $k_2 < 1$). This is also true for the "pressure" p [see (2.3)]. Actually, the function

$$g(V) = \varphi_2/J = 3V/(2 + \sqrt{4 - 3h^2V^2}) \qquad (V = 1/J)$$

has the derivative $g'(V) > 0$. It therefore follows from (2.12) that $\hat{p} > \hat{p}_\infty$ for $1 > \hat{\varphi}_\infty > 2/5$.

Note that the relation $\hat{\varphi} = (k_2 - 1 + 2k_2\hat{\varphi}_\infty)/2 = (4 - \hat{\varphi}_\infty)/(1 + 20\hat{\varphi}_\infty)$ results from (2.12′), i.e., for $1 > \hat{\varphi}_\infty > 2/5$, we have

$$1/7 < \hat{\varphi} < 2/5 \quad (9/49 < \hat{\lambda} < 12/25). \tag{2.13}$$

3. Linearization of the Equations of Radiation Hydrodynamics. Formulation of the Problem of the Stability of Shock Waves.

We consider the following constant solution of system (1.1′) and (1.2′) for $B \equiv 0$:

$$\mathbf{U} = \hat{\mathbf{U}} = \begin{pmatrix} \hat{J} \\ \hat{H}^1 \\ 0 \\ 0 \end{pmatrix},$$

where $\hat{J} = \text{const} > 0$, $\hat{H}^1 = \text{const}$ (we assume, for definiteness, that $\hat{H}^1 > 0$), and $\hat{\Lambda}^1 = \hat{H}^1/\hat{J} < 1$. Linearizing system (1.1′) and (1.2′) with respect to this solution, we obtain a linear system of equations with constant coefficients [see system (1.19)]:

$$B^{(0)}(\hat{\mathbf{U}})\frac{\partial \mathbf{U}}{\partial t} + \sum_{k=1}^{3} B^{(k)}(\hat{\mathbf{U}})\frac{\partial \mathbf{U}}{\partial x^k} = 0. \tag{3.1}$$

Here the vector $\mathbf{U}$ consists of small perturbations of the components of the initial vector $\mathbf{U}$ (which is denoted by the same letter).

We now determine the eigenvalues of the matrix $B^{(1)}(\hat{\mathbf{U}})$ or, more exactly, $A^{(1)}(\hat{\mathbf{U}})$. Since $B^{(1)}(\hat{\mathbf{U}}) = (1/\hat{m})B^{(0)}(\hat{\mathbf{U}})A^{(1)}(\hat{\mathbf{U}})B^{(0)}(\hat{\mathbf{U}})$ and $B^{(0)}(\hat{\mathbf{U}}) > 0$, we have $\hat{m} = m(\hat{\mathbf{U}}) > 0$ (see Sec. 1). After simple calculations we see that the matrix $A^{(1)}(\hat{\mathbf{U}})$ has eigenvalues

$$\lambda_{1,2} = \frac{4}{3}\frac{\hat{J}\hat{\varphi}_1\hat{r}_1}{\hat{G}^2}, \quad \lambda_{3,4} = \frac{4}{\hat{G}^4}\left\{2\hat{r}_1(\hat{r}_1^2 + 2\hat{r}_0^2) \pm \sqrt{\hat{r}_1^6 + \hat{r}_0^6 + 23\hat{r}_0^2\hat{r}_1^4 + 11\hat{r}_0^4\hat{r}_1^2}\right\}, \tag{3.2}$$

where $\hat{r}_0 = -\hat{J}/(\hat{\varphi}_2\hat{d})$; $\hat{r}_1 = \hat{H}^1/\hat{d}$; and $\hat{d} = \sqrt{(4\hat{J}/\hat{\varphi}_2)(\hat{J}\hat{\varphi}_1/3)^{3/2}}$; if $\hat{\varphi} > 1/\sqrt{3}$, then $\lambda_{1,2,3,4} > 0$ and if $\hat{\varphi} < 1/\sqrt{3}$, then $\lambda_{1,2,3} > 0$ and $\lambda_4 < 0$.

Note that system (3.1) can be rewritten in the form [which is easy to obtain by linearization of nondivergent system (2.5) and (2.6)]

$$\begin{gathered} Lp - c_0\xi_1 p + c_1\{c_2\xi_1 H^1 + \xi_2 H^2 + \xi_3 H^3\} = 0, \\ LH^1 + b_0 Lp + b_1\xi_1 p = 0, \quad LH^2 + \xi_2 p = 0, \quad LH^3 + \xi_3 p = 0. \end{gathered} \tag{3.1′}$$

Here p is a small perturbation of the "pressure" [see (2.3)];

$$L = \tau + \frac{\hat{\varphi}}{\sqrt{\hat{\lambda}}}\xi_1; \quad \tau = \frac{\partial}{\partial t}; \quad \xi_k = \frac{\partial}{\partial x^k} \quad (k = 1,2,3); \quad c_0 = \frac{2(\hat{\sigma} - 1)(4 + \hat{\lambda} - 2\hat{\sigma})}{\sqrt{\hat{\lambda}}\,\hat{\sigma}(4 - \hat{\sigma})};$$

$$c_1 = \frac{2}{3}\frac{\hat{\sigma} - 1}{\hat{\sigma}}; \quad c_2 = 2\frac{\hat{\sigma} - 1}{4 - \hat{\sigma}}; \quad b_0 = -\frac{3}{2}\frac{\hat{\sigma}(2 - \hat{\sigma})}{\sqrt{\hat{\lambda}}\,(\hat{\sigma} - 1)}; \quad b_1 = 2\frac{4 - \hat{\lambda} - 2\hat{\sigma}}{\hat{\lambda}}.$$

It is evident that system (3.1′) is similar to the system of acoustic equations in ordinary gas dynamics [5, 7]. As in gas dynamics, in our case the function p also satisfies the wave equation $L^2 p - d_0 L\xi_1 p - c_1\left\{d_1\xi_1^2 p + \xi_2^2 p + \xi_3^2 p\right\} = 0$, where $(d_0 = 4(\hat{\sigma} + \hat{\lambda} - 2)/(\sqrt{\hat{\lambda}}\hat{\sigma})$ and $d_1 = b_1 c_2$. Under the additional assumption that $\hat{\varphi} < 2/5$ (i.e., $\hat{\sigma} > 8/5$), the latter equation can be rewritten as

$$\{(\tau')^2 - (\xi_1')^2 - \xi_2^2 - \xi_3^2\}p = 0, \tag{3.3}$$

where the new differential operators τ' and ξ_1' are defined as $\tau = \hat{p}_0\tau'$ and $\xi_1 = \hat{p}_1\xi_1' + \hat{p}_2\tau'$, where $\hat{p}_0 =$

$\sqrt{(5\hat{\sigma}-8)/(6(\hat{\sigma}-1))}$, $\hat{p}_1 = 1/\sqrt{3}\hat{p}_0$, and $\hat{p}_2 = \sqrt{\hat{\lambda}}/(2(\hat{\sigma}-1)\hat{p}_0)$.

We linearize system (1.1′) and (1.2′) with respect to the piecewise constant solution described in Sec. 2. As a result, we have a mathematical formulation of the problem of the stability of shock waves in the studied model of radiation hydrodynamics.

Basic Problem I is to find piecewise smooth functions that are solutions of system (3.1) (for $t > 0$ and $\mathbf{x} \in R^3_+$) and of the system

$$B^{(0)}(\hat{\mathbf{U}}_\infty)\frac{\partial \mathbf{U}}{\partial t} + \sum_{k=1}^{3} B^{(k)}(\hat{\mathbf{U}}_\infty)\frac{\partial \mathbf{U}}{\partial x^k} = 0 \tag{3.4}$$

(for $t > 0$ and $\mathbf{x} \in R^3_-$) and satisfy the boundary conditions

$$\left[\frac{1}{\sqrt{\hat{\lambda}}}\right]F_t = [H^1], \quad \left[\frac{8-5\hat{\sigma}}{4-\hat{\sigma}}p\right] = \left[\frac{3\sqrt{\hat{\lambda}}}{4-\hat{\sigma}}H^1\right], \quad \left[\frac{\hat{\varphi}}{\sqrt{\hat{\lambda}}}\right]F_{x^k} + \left[\frac{\hat{\varphi}}{\sqrt{\hat{\lambda}}}H^k\right] = 0, \quad k = 2,3 \tag{3.5}$$

(for $t > 0$, $\mathbf{x}' \in R^2$, and $x^1 = 0$) and the initial data

$$\mathbf{U}(0,\mathbf{x}) = \mathbf{U}_0(\mathbf{x}), \quad \mathbf{x} \in R^3_\pm, \quad F(0,\mathbf{x}') = F_0(\mathbf{x}'), \quad \mathbf{x}' \in R^2 \tag{3.6}$$

for $t = 0$. Here $F = \hat{H}^1\delta f$; $\delta f(t,\mathbf{x}')$ is a small displacement of the discontinuity front; and $R^3_\pm = \{\mathbf{x} \,|\, x^1 \lessgtr 0,\ \mathbf{x}' \in R^2\}$;

$$\hat{\mathbf{U}}_\infty = \begin{pmatrix} \hat{J}_\infty \\ \hat{H}^1_\infty \\ 0 \\ 0 \end{pmatrix}; \quad \hat{H}^1 = \hat{H}^1_\infty \quad \text{[see relation (2.11)]};$$

$$\left[\frac{1}{\sqrt{\hat{\lambda}}}\right] = \frac{1}{\sqrt{\hat{\lambda}}} - \frac{1}{\sqrt{\hat{\lambda}_\infty}}; \qquad \hat{\lambda}_\infty = \frac{(\hat{H}^1)^2}{\hat{J}^2_\infty},$$

etc. Taking into account formulas (3.2), we assume that the inequality

$$\hat{\varphi}_\infty > 1/\sqrt{3} \tag{3.7}$$

is satisfied. Note that in this case, *a priori*, $\hat{\varphi}_\infty > 2/5$ and $\hat{\varphi} < (4\sqrt{3}-1)/(20+\sqrt{3}) < 2/5 < 1/\sqrt{3}$ [see (2.13)]. Thus, with satisfaction of condition (3.7), all eigenvalues of the matrix $A^{(1)}(\hat{\mathbf{U}}_\infty)$ are positive. Hence, system (3.4) does not require boundary conditions for $x^1 = 0$. Then, without loss of generality, we assume that $\mathbf{U}(t,\mathbf{x}) \equiv 0$ for $x^1 < 0$. Similar reasoning with allowance for the inequality $\hat{\varphi} < 1/\sqrt{3}$ shows that system (3.1) requires three boundary conditions and one boundary condition for determining the function $F(t,\mathbf{x}')$. So, there are as many boundary conditions (3.5) as are required for the discontinuity to be evolutionary [8] if condition (3.7), which is called the evolutionary condition, is satisfied.

Allowing for the above remarks, we reformulate the basic problem I [with satisfaction of condition (3.7)].

Basic Problem II is to find a solution of system (3.1) for $t > 0$ and $\mathbf{x} \in R^3_+$ that satisfies the boundary conditions

$$F_t = \mu p, \quad H^1 + dp = 0, \quad H^{2,3} - (\tilde{\lambda}/\mu)F_{x^{2,3}} = 0 \tag{3.5′}$$

for $t > 0$, $\mathbf{x}' \in R^2$, and $x^1 = 0$ and the initial data

$$\mathbf{U}(0,\mathbf{x}) = \mathbf{U}_0(\mathbf{x}), \quad \mathbf{x} \in R^3_+, \quad F(0,\mathbf{x}') = F_0(\mathbf{x}'), \quad \mathbf{x}' \in R^2 \tag{3.6′}$$

for $t = 0$. Here $\mu = \sqrt{\hat{\lambda}_\infty}(8-5\hat{\sigma})/\big(3(\sqrt{\hat{\lambda}_\infty} - \sqrt{\hat{\lambda}})\big)$; $d = (5\hat{\sigma}-8)/3\sqrt{\hat{\lambda}}$; and $\tilde{\lambda} = (8-5\hat{\sigma})/6\hat{\varphi}$.

4. Stability of a Shock Wave. We describe the process of deriving an *a priori* estimate of the solution without loss of smoothness of basic problem II in a plane case. In view of replacement (2.1), this estimate leads to an *a priori* estimate with decreasing for the solution of the mixed problem obtained by linearization of initial system (1.1) and (1.2) in the plane case (for $B \equiv 0$) with respect to the piecewise smooth solution described above [see (2.2)]. And finally, from this estimate with decreasing follows the convergence of the solution of the mixed problem for linearized system (1.1) and (1.2) to a trivial solution in the norm $W_2^2(R_+^2)$ for $t \to \infty$.

To obtain an *a priori* estimate for the solution of basic problem II, we construct an extended system [5] for determining the second derivatives of the components of the solution. The process of construction involves two stages. In the first stage, using system (3.1) (in the case of plane symmetry) we set up the following symmetric t-hyperbolic (after Friedrichs) system:

$$B_p^{(0)}\frac{\partial \mathbf{U}_p}{\partial t} + \sum_{k=1}^{2} B_p^{(k)}\frac{\partial \mathbf{U}_p}{\partial x^k} = 0. \tag{4.1}$$

Here $\mathbf{U}_p = (\tau^2\mathbf{U}^*, \tau\xi_1\mathbf{U}^*, \tau\xi_2\mathbf{U}^*, \xi_1^2\mathbf{U}^*, \xi_1\xi_2\mathbf{U}^*, \xi_2^2\mathbf{U}^*)^*$ and $B_p^{(\alpha)} = \operatorname{block\,diag}(B^{(\alpha)}, B^{(\alpha)}, B^{(\alpha)}, B^{(\alpha)}, B^{(\alpha)}, B^{(\alpha)})$ $(\alpha = \overline{0,2})$ are block diagonal matrices.

Writing the energy integral for symmetric system (4.1) in differential form [5] and integrating it over the domain R_+^2, we obtain

$$\frac{d}{dt} I_0(t) - \int\limits_{R^1} (B_p^{(1)}\mathbf{U}_p, \mathbf{U}_p)\Big|_{x^1=0} dx^2 = 0, \tag{4.2}$$

where

$$I_0(t) = \iint\limits_{R_+^2} (B_p^{(0)}\mathbf{U}_p, \mathbf{U}_p)d\mathbf{x}; \quad \mathbf{x} = (x^1, x^2); \quad R_+^2 = \{\mathbf{x}\,|\,x^1 > 0,\ x^2 \in R^1\}.$$

In deriving (4.2) we assume that $|\mathbf{U}_p| \to 0$ for $x^1 \to \infty$ or $|x^2| \to \infty$.

Estimating the second term in equality (4.2) by means of boundary conditions (3.5′) and system (3.1′) (in the case of plane symmetry), for $x^1 = 0$, we have

$$\frac{d}{dt} I_0(t) - M_1 \int\limits_{R^1} P\Big|_{x^1=0} dx^2 \leqslant 0, \tag{4.3}$$

where $M_1 > 0$ is a constant and $P = p_{tt}^2 + p_{tx^1}^2 + p_{tx^2}^2 + p_{x^1x^1}^2 + p_{x^1x^2}^2 + p_{x^2x^2}^2$.

Let us go to the second stage of constructing the extended system. We rewrite Eq. (3.3) for the case of plane symmetry:

$$(L_1^2 - L_2^2 - L_3^2)p = 0. \tag{4.4}$$

Here $L_1 = \tau'$, $L_2 = \xi_1'$, and $L_3 = \xi_2$. If the function $p(t, \mathbf{x})$ satisfies Eq. (4.4), the vector $\mathbf{W} = (\mathbf{Y}_1^*, \mathbf{Y}_2^*, \mathbf{Y}_3^*)^*$ [$\mathbf{Y}_1 = L_1\mathbf{Y}$, $\mathbf{Y}_2 = L_2\mathbf{Y}$, $\mathbf{Y}_3 = L_3\mathbf{Y}$, $\mathbf{Y} = \widetilde{\nabla} p$, and $\widetilde{\nabla} = (L_1, L_2, L_3)^*$] satisfies a system of the form [5, 8]

$$\{\hat{A}L_1 - \hat{B}L_2 - \hat{C}L_3\}\mathbf{W} = 0, \tag{4.5}$$

where

$$\hat{A} = \begin{pmatrix} \mathcal{K} & \mathcal{L} & \mathcal{M} \\ \mathcal{L} & \mathcal{K} & i\mathcal{N} \\ \mathcal{M} & -i\mathcal{N} & \mathcal{K} \end{pmatrix}; \quad \hat{B} = \begin{pmatrix} \mathcal{L} & \mathcal{K} & i\mathcal{N} \\ \mathcal{K} & \mathcal{L} & \mathcal{M} \\ -i\mathcal{N} & \mathcal{M} & -\mathcal{L} \end{pmatrix}; \quad \hat{C} = \begin{pmatrix} \mathcal{M} & -i\mathcal{N} & \mathcal{K} \\ i\mathcal{N} & -\mathcal{M} & \mathcal{L} \\ \mathcal{K} & \mathcal{L} & \mathcal{M} \end{pmatrix};$$

$\mathcal{K}$, $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ are now arbitrary Hermitian matrices of order 3. Reverting to the differential operators τ, ξ_1, and ξ_2 in system (4.5), we obtain

$$\{D\tau - \hat{B}\xi_1 - \hat{p}_1\hat{C}\xi_2\}\mathbf{W} = 0 \quad \Big(D = \frac{\hat{p}_1}{\hat{p}_0}(\hat{A} + M\hat{B}), \quad M = \frac{\hat{p}_2}{\hat{p}_1} = \frac{\sqrt{3\hat{\lambda}}}{2(\hat{\sigma} - 1)}(< 1)\Big). \tag{4.6}$$

Note that the following relationships are valid [5, 8]:

$$\hat{A} = T_0^*\{I_2\times\widetilde{H}\}T_0, \quad \hat{B} = T_0^*\left\{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\times\widetilde{H}\right\}T_0, \quad \hat{C} = T_0^*\left\{\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\times\widetilde{H}\right\}T_0. \tag{4.7}$$

Here

$$T_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\times I_3; \quad \widetilde{H} = \begin{pmatrix} \mathcal{K}-\mathcal{M} & -\mathcal{L}-i\mathcal{N} \\ -\mathcal{L}+i\mathcal{N} & \mathcal{K}+\mathcal{M} \end{pmatrix};$$

$I_2\times\widetilde{H}$ is the Kronecker product of matrices I_2 and $\widetilde{H}$, etc.; I_2 is a unit matrix of order 2, etc. By virtue of (4.7), we have

$$D = \frac{\hat{p}_1}{\hat{p}_0}T_0^*\left\{\begin{pmatrix} 1 & -M \\ -M & 1 \end{pmatrix}\times\widetilde{H}\right\}T_0. \tag{4.8}$$

To find boundary conditions for system (4.6), following [5], we find the scalar product of system (3.1′) (in the case of plane symmetry) and the vector $(\tau, -c_1c_2\tau, 0, 0)^*$. Considering the obtained expression for $x^1 = 0$, by means of boundary conditions (3.5′), we have the relation

$$m(L_2^2 + L_3^2)p + nL_2^2p - \gamma L_1L_2p = 0, \quad x^1 = 0, \tag{4.9}$$

where $m = (5\hat{\sigma}-8)(13-7\hat{\sigma})/(27\sqrt{\hat{\lambda}}\hat{\sigma})$, $n = (5\hat{\sigma}-8)(\hat{\sigma}-1)/(9\sqrt{\hat{\lambda}}\hat{\sigma})$, and $\gamma = (5\hat{\sigma}-8)/(3\sqrt{3}\hat{\sigma})$.

Allowing for (4.4) and (4.9) for $x^1 = 0$, as boundary conditions for system (4.6) we use the expressions [5] $L_1(L_1p) - L_2(L_2p) - L_3(L_3p) + \alpha\{L_1(L_2p) - L_2(L_1p)\} = 0$, $L_3(L_2p) - L_2(L_3p) = 0$, and $L_1(L_2p) - ((m+n)/\gamma)\, L_2(L_2p) - (m/\gamma)\, L_3(L_3p) = 0$, which can be rewritten as

$$A_1\mathbf{Y}_1 + B_1\mathbf{Y}_2 + C_1\mathbf{Y}_3 = 0. \tag{4.10}$$

Here

$$A_1 = \begin{pmatrix} 1 & \alpha & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} -\alpha & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -\dfrac{m+n}{\gamma} & 0 \end{pmatrix}; \quad C_1 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -\dfrac{m}{\gamma} \end{pmatrix}, \quad \frac{m+n}{\gamma} = \frac{2}{9}(5-2\hat{\sigma})\sqrt{\frac{3}{\hat{\lambda}}},$$

and $\alpha > 1$ is a constant. Let $\mathbf{Z} = \begin{pmatrix} \mathbf{Z}_{\rm I} \\ \mathbf{Z}_{\rm II} \end{pmatrix} = T_0\mathbf{W}$, where $\mathbf{Z}_{\rm I} = \begin{pmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{pmatrix}$ and $\mathbf{Z}_{\rm II} = \begin{pmatrix} \mathbf{Z}_3 \\ \mathbf{Z}_4 \end{pmatrix}$ [$\mathbf{Z}_k$ $(k = \overline{1,4})$ are vectors of dimension 3] Since $\mathbf{Y}_1 = (\sqrt{2}/2)\,(\mathbf{Z}_1 + \mathbf{Z}_4)$, $\mathbf{Y}_2 = -\sqrt{2}\mathbf{Z}_2 = -\sqrt{2}\mathbf{Z}_3$, and $\mathbf{Y}_3 = (\sqrt{2}/2)(\mathbf{Z}_4 - \mathbf{Z}_1)$, conditions (4.10) can be written as

$$\mathbf{Z}_{\rm I} = G\mathbf{Z}_{\rm II}. \tag{4.11}$$

Here

$$G = \begin{pmatrix} G_1 & -G_2 \\ I_3 & 0 \end{pmatrix}; \quad G_1 = 2(A_1 - C_1)^{-1}B_1; \quad G_2 = (A_1 - C_1)^{-1}(A_1 + C_1).$$

Let all eigenvalues of the matrix G lie strictly in the left half-plane, i.e., Re $\lambda_j(G) < 0$ $(j = \overline{1,6})$. The latter is valid for the inequalities $m > 0$ and $n > 0$, which are satisfied by virtue of (2.13), because $8/5 < \hat{\sigma} < 13/7$. We now set up the Lyapunov matrix equation

$$G^*\widetilde{H} + \widetilde{H}G = -G_0 \tag{4.12}$$

to find the matrix $\widetilde{H}$, which appears in formulas (4.7). As is known [9], Eq. (4.12) has the unique solution

$$\widetilde{H} = \begin{pmatrix} \widetilde{H}_1 & \widetilde{H}_2 \\ \widetilde{H}_2^* & \widetilde{H}_3 \end{pmatrix} > 0, \quad \widetilde{H}_1 = \widetilde{H}_1^*, \quad \widetilde{H}_3 = \widetilde{H}_3^*$$

for any real symmetric positive-definite matrix G_0. In this case, the matrix $\widetilde{H}$ is also real symmetric, and matrices $\mathcal{K}$, $\mathcal{L}$, $\mathcal{M}$, and $\mathcal{N}$ are found as follows:

$$\mathcal{K} = \frac{1}{2}(\widetilde{H}_1 + \widetilde{H}_3), \quad \mathcal{M} = \frac{1}{2}(\widetilde{H}_3 - \widetilde{H}_1), \quad \mathcal{L} = -\frac{1}{2}(\widetilde{H}_2 + \widetilde{H}_2^*), \quad i\mathcal{N} = \frac{1}{2}(\widetilde{H}_2^* - \widetilde{H}_2).$$

Since $\widetilde{H} > 0$ we have $D > 0$ by virtue of (4.8).

For system (4.6), we write the energy integral in differential form and integrate the resulting identity over the domain R^2_+, assuming that $|\mathbf{W}| \to 0$ as $x^1 \to \infty$ or $|x^2| \to \infty$. As a result, we have

$$\frac{d}{dt} I_1(t) + \int_{R^1} (\hat{B}\mathbf{W}, \mathbf{W})\Big|_{x^1=0} dx^2 = 0 \quad \Big(I_1(t) = \iint_{R^2_+} (D\mathbf{W}, \mathbf{W}) d\mathbf{x}\Big). \tag{4.13}$$

According to (4.7) and (4.11), the quadratic form is

$$(\hat{B}\mathbf{W}, \mathbf{W})\Big|_{x^1=0} = (G_0\mathbf{Z}_{\mathrm{II}}, \mathbf{Z}_{\mathrm{II}}) > 0. \tag{4.14}$$

Since

$$\mathbf{Z}_{\mathrm{II}} = \frac{\sqrt{2}}{2}\begin{pmatrix} -\mathbf{Y}_2 \\ \mathbf{Y}_1 + \mathbf{Y}_3 \end{pmatrix},$$

we have

$$(G_0\mathbf{Z}_{\mathrm{II}}, \mathbf{Z}_{\mathrm{II}})\Big|_{x^1=0} > M_2\big\{(L_1^2 p)^2 + (L_1L_2p)^2 + (L_1L_3p)^2 + (L_2^2p)^2 + (L_2L_3p)^2 + (L_3^2p)^2\big\}\Big|_{x^1=0} > \widetilde{M}_2 P\Big|_{x^1=0}, \tag{4.15}$$

where M_2 and $\widetilde{M}_2 > 0$ are the constants specified by the norm of the matrix G_0. Taking (4.14) and (4.15) into account, from (4.13) we have

$$\frac{d}{dt} I_1(t) + \widetilde{M}_2 \int_{R^1} P\Big|_{x^1=0} dx^2 < 0. \tag{4.16}$$

Summing inequalities (4.3) and (4.16) and bearing in mind that a proper choice of the matrix G_0 [i.e., of the constant $\widetilde{M}_2$ (4.15)] makes it possible to attain positive definiteness of the form

$$(\widetilde{M}_2 - M_1)\widetilde{P}\Big|_{x_1=0}, \tag{4.17}$$

we obtain

$$\frac{d}{dt} I_2(t) < 0, \quad t > 0 \quad (I_2(t) = I_0(t) + I_1(t)),$$

which leads to an *a priori* estimate for the second derivatives of the solution of the basic problem II:

$$I_2(t) < I_2(0), \quad t > 0. \tag{4.18}$$

To estimate the solution $\mathbf{U}$ itself and its first derivatives, we set up an extended system of (4.1), (4.6), and the following system (which is a set of obvious relations): $\partial \mathbf{V}_p/\partial t - \tau\mathbf{V}_p = 0$, where $\mathbf{V}_p = (\mathbf{U}^*, \tau\mathbf{U}^*, \xi_1\mathbf{U}^*, \xi_2\mathbf{U}^*)^*$. Writing the energy integral for this system and taking (4.18) into account, we obtain

$$\frac{d}{dt} I(t) < 2\iint_{R^2_+} (\tau\mathbf{V}_p, \mathbf{V}_p)\, d\mathbf{x} \quad \Big(I(t) = I_2(t) + \iint_{R^2_+} (\mathbf{V}_p, \mathbf{V}_p)\, d\mathbf{x}\Big). \tag{4.19}$$

Estimating the right-hand side of (4.19) by the Holder inequality, we arrive at the inequality

$$\frac{d}{dt} I(t) < 2(I(0)I(t))^{1/2}, \quad t > 0,$$

from which follows the desired *a priori* estimate for the solution of basic problem II:

$$I(t) < I(0)(t+1)^2, \quad t > 0. \tag{4.20}$$

Finally, for the solution of basic problem II, from (4.20) follows the estimate

$$\|\mathbf{U}(t)\|_{W_2^2(R_+^2)} \leqslant M_3, \quad 0 < t \leqslant T < \infty, \tag{4.21}$$

where $M_3 < \infty$ is a positive constant specified by T. Note that, as in [5, 8, 10], for the function $F(t, x^2)$ we can derive the estimate

$$\|F\|_{W_2^3((0,T)\times R^1)} \leqslant M_4, \tag{4.22}$$

where $M_4 < \infty$ is a positive constant specified by T. In deriving estimate (4.22) we use the fact that quadratic form (4.17) is positive definite.

Estimates (4.21) and (4.22) show that basic problem II in the case of plane symmetry is well posed. Note also that, using another (more cumbersome) procedure of constructing an extended system [5, 11], one can also obtain *a priori* estimates of the solution of basic problem II in the general (three-dimensional) case.

With allowance for substitution (2.1), estimate (4.20) takes the form

$$I(t) < I(0)(t+1)^2 e^{-2t}, \quad t > 0, \tag{4.23}$$

where the vector $\mathbf{U}$, which appears indirectly in the formula for the aggregate $I(t)$, now denotes the vector of small perturbations of the solution of the mixed problem for initial linearized system (1.1) and (1.2). Then it follows from the estimate with decreasing (4.23) that the solution $\mathbf{U}(t, \mathbf{x})$ converges to the trivial solution in the norm $W_2^2(R_+^2)$ for $t \to \infty$, i.e., $\lim_{t\to\infty} \|\mathbf{U}(t)\|_{W_2^2(R_+^2)} = 0$.

Thus, shock waves are stable in the investigated model of radiation hydrodynamics.

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